

# Diocese of Joliet: Standards for Mathematics Curriculum, Grade 3

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# *Basic Principles underlying All Standards to be used for the Planning of Curriculum for the Diocese of Joliet*

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Basic principles which inform all Catholic education in the Schools of the Diocese of Joliet are:

- **Human knowledge is a truth to be discovered.**
- All knowledge, in some way, reflects God's Truth, Beauty and Goodness.
- Curriculum and instruction enable deeper incorporation of the children into the Church, the formation of community within the school; and respect for the uniqueness and dignity of each person as created in the image of God.
- Education fosters growth in Christian virtue and contributes to development and formation of the whole person in light of his/her ultimate end and the good of the society of which he/she is a member.
- Each subject is to be examined in the context of the Catholic faith and is to be illuminated by Gospel values.
- Learning and formation occur in the Catholic school without separation as does the development of each student on both the natural and supernatural levels.
- Curriculum and instruction seeks to promote a synthesis of faith, life and culture and to form students as disciples of Jesus.



**DIOCESE OF JOLIET  
CATHOLIC SCHOOL**



**STANDARDS FOR MATHEMATICS**

Mathematics is the study of quantity, structure, space, and change. Attention should be paid to the needs of today's society in teaching mathematics by fostering real world application, enabling students to undertake responsibilities in society both locally and globally while witnessing to the faith.

Individual subjects must be taught according to their own particular methods. It would be wrong to consider subjects as mere adjuncts to faith or as a useful means of teaching apologetics. They enable the pupil to assimilate skills, knowledge, intellectual methods and moral and social attitudes, all of which help to develop his personality and lead him to take his place as an active member of the community of man. Their aim is not merely the attainment of knowledge but the acquisition of values and the discovery of truth. *The Catholic School, 39*

**In a Catholic school, curricular formation...**

1. Involves the integral formation of the whole person, body, mind, and spirit, in light of his or her ultimate end and the good of society.<sup>i</sup>
2. Promotes human virtues and the dignity of the human person, as created in the image and likeness of God and modeled on the person of Jesus Christ.<sup>ii</sup>
3. Seeks to know and understand objective reality which includes transcendent Truth, is knowable by reason and faith, and finds its origin, unity, and end in God.
4. Develops a Catholic worldview and enables a deeper incorporation of the student into the heart of the Catholic Church.<sup>iii</sup>
5. Encourages a synthesis of faith, life, and culture.<sup>iv</sup>

## THIRD GRADE MATHEMATICS STANDARDS

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Use mathematical vocabulary consistent with Diocesan expectations.

### SEE APPENDIX A: STANDARDS FOR MATHEMATICAL PRACTICE

#### CATHOLIC STANDARDS:

- **Develop the mental habits of precise, determined, careful, and accurate questioning, inquiry, and reasoning. CSGS1**
- **Develop lines of inquiry (as developmentally appropriate) to understand why things are true and why they are false. CSGS2**
- **Recognize the power of the human mind as both a gift from God and a reflection of Him in whose image and likeness we are made. CSGS3**
- **Survey the truths about mathematical objects that are interesting in their own right and independent of human opinions. CSGS4**

#### OA - Operations and Algebraic Thinking

##### **Represent and solve problems involving multiplication and division.**

1. Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*
2. Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \_ \div 3$ ,  $6 \times 6 = ?$ .*

##### **Understand properties of multiplication and the relationship between multiplication and division.**

5. Apply properties of operations as strategies to multiply and divide. Examples: If  $6 \times 4 = 24$  is known, then  $4 \times 6 = 24$  is also known. (Commutative property of multiplication.)  $3 \times 5 \times 2$  can be found by  $3 \times 5 = 15$  then  $15 \times 2 = 30$ , or by  $5 \times 2 = 10$  then  $3 \times 10 = 30$ . (Associative property of multiplication.) Knowing that  $8 \times 5 = 40$  and  $8 \times 2 = 16$ , one can find  $8 \times 7$  as  $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$ . (Distributive property.) Know that any number times 0 is 0. (Students need not use formal terms for these properties.)
6. Understand division as an unknown-factor problem. *For example, divide  $32 \div 8$  by finding the number that makes 32 when multiplied by 8.*

##### **Multiply and divide within 100.**

7. Fluently multiply and divide within 100 (facts through 10) using strategies such as the relationship between multiplication and division (e.g., knowing that  $8 \times 5 = 40$ , one knows  $40 \div 5 = 8$ ) or properties of operations. By the end of Grade 3, know from memory all products of one-digit numbers.

**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).)
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

**NBT - Number and Operations in Base Ten**

**Use place value understanding and properties of operations to perform multi-digit arithmetic.**

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (A range of algorithms may be used.)
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g.,  $9 \times 80$ ,  $5 \times 60$ ) using strategies based on place value and properties of operations. (A range of algorithms may be used.)

**NF - Number and Operations – Fractions**

(Grade 3 expectations are limited to fractions with denominators 2, 3, 4, 6, and 8)

**Develop understanding of fractions as numbers.**

1. Understand a fraction  $1/b$  as the quantity formed by 1 part when a whole is partitioned into  $b$  equal parts; understand a fraction  $a/b$  as the quantity formed by  $a$  parts of size  $1/b$ .
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)
  - a. Represent a fraction  $1/b$  on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into  $b$  equal parts. Recognize that each part has size  $1/b$  and that the endpoint of the part based at 0 locates the number  $1/b$  on the number line.
  - b. Represent a fraction  $a/b$  on a number line diagram by marking off  $a$  lengths  $1/b$  from 0. Recognize that the resulting interval has size  $a/b$  and that its endpoint locates the number  $a/b$  on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
  - a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)
  - b. Recognize and generate simple equivalent fractions (e.g.,  $1/2 = 2/4$ ,  $4/6 = 2/3$ ), Explain why the fractions are equivalent, e.g., by using a visual fraction model.
  - c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form  $3 = 3/1$ ; recognize that  $6/1 = 6$ ; locate  $4/4$  and 1 at the same point of a number line diagram.

- d. Compare two fractions with the same numerator or the same denominator, by reasoning about their size, Recognize that valid comparisons rely on the two fractions referring to the same whole. Record the results of comparisons with the symbols  $>$ ,  $=$ , or  $<$ , and justify the conclusions, e.g., by using a visual fraction model.

### **MD - Measurement and Data**

#### **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as  $\text{cm}^3$  and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems (problems involving notions of “times as much.”))

#### **Represent and interpret data.**

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets. Use tallies, charts and/or tables to display and interpret information.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

#### **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
  - a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
  - b. A plane figure which can be covered without gaps or overlaps by  $n$  unit squares is said to have an area of  $n$  square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
  - a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
  - b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
  - c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths  $a$  and  $b + c$  is the sum of  $a \times b$  and  $a \times c$ . Use area models to represent the distributive property in mathematical reasoning.
  - d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

#### **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different area or with the same area and different perimeter.

## **G- Geometry**

### **Reason with shapes and their attributes.**

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.
2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part is  $\frac{1}{4}$  of the area of the shape.*

## APPENDIX A: STANDARDS FOR MATHEMATICAL PRACTICES AND CATHOLIC STANDARDS DISPOSITIONS

### STANDARDS FOR MATHEMATICAL PRACTICES AND CATHOLIC DISPOSITIONS

The Standards for Mathematical Practices and Dispositions describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. Students develop an understanding of the ability of the human intellect to know and the desire of the will to want to know more (CSDS9). As students progress in their mathematical knowledge, they will evaluate the ongoing nature of mathematical inquiry, its inexhaustibility, and its opening to the infinite (CSIS6). These practices rest on important “processes and proficiencies” with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition.

#### **1. Make sense of problems and persevere in solving them.**

CSDS3: Show interest in the pursuit of understanding for its own sake.

CSDS4: Exhibit joy at solving difficult mathematical problems and operations.

Students Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

#### **2. Reason abstractly and quantitatively.**

CSDS8: Exhibit humility at knowing that as a human being, man can only grasp a portion of the truths of the universe (Grades 7 & 8).

Mathematically proficient students make sense of the quantities and their relationships in problem situations. Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative

reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

### **3. Construct viable arguments and critique the reasoning of others.**

**CCSS5:** Demonstrate an interest of the mental processes within mathematics (order, perseverance, and logical reasoning) and how these processes help develop natural virtues (self-discipline and fortitude).

**CCSS2:** Demonstrate how sound logical arguments are foundational to mathematics.

**CCSS6:** Propose how mathematical objects or relationships (such as the golden mean, the Fibonacci numbers, the musical scale, geometric proofs) suggest divine origin (Grades 7 & 8).

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

### **4. Model with mathematics.**

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

### **5. Use appropriate tools strategically.**

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze

graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## **6. Attend to precision.**

**CSDS5:** Demonstrate an interest of the mental processes within mathematics (order, perseverance, and logical reasoning) and how these processes help develop natural virtues (self-discipline and fortitude).

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## **7. Look for and make use of structure.**

**CSDS1:** Display a sense of wonder about mathematical relationships as well as confidence in mathematical certitude.

**CSDS2:** Respond to the beauty, harmony, proportion, radiance, and wholeness present in mathematics.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see  $7 \times 8$  equals the well-remembered  $7 \times 5 + 7 \times 3$ , in preparation for learning about the distributive property. In the expression  $x^2 + 9x + 14$ , older students can see the 14 as  $2 \times 7$  and the 9 as  $2 + 7$ . They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see  $5 - 3(x - y)^2$  as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers  $x$  and  $y$ .

## **8. Look for and express regularity in repeated reasoning.**

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation  $(y - 2)/(x - 1) = 3$ . Noticing the regularity in the way terms cancel when expanding  $(x - 1)(x + 1)$ ,  $(x - 1)(x^2 + x + 1)$ , and  $(x - 1)(x^3 + x^2 + x + 1)$  might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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<sup>i</sup> *The Catholic School*, 1977, #36, 47, 49. *Gravissimum Educationis*, 1965, #1, par. 1; USCCB. *Seven themes of Catholic social teaching*.

<sup>ii</sup> *The Religious Dimension of Education in a Catholic School*, 1988, #52, 56; *The Catholic School*, 1977, #55.

<sup>iii</sup> *The Religious Dimension of Education in a Catholic School*, 1988, #71, 74-77; *The Catholic School*, 1977, #50

<sup>iv</sup> *The Religious Dimension of Education in a Catholic School*, 1988, #52; *The Catholic School*, #37.